Efficient Route Planning

SS 2011

Lecture 4, Friday June 4th, 2011

(Arc flags)

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Overview of this lecture

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- Feedback from the exercises
  - Your experimental results
  - Your experiences
- A new algorithm
  - Arc flags
  - Idea, Analysis, Improvement
- Exercises
  - Implement arc flags and do experiments

Experimental results from Ex. Sheet 1

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- See the table on the Wiki
  - Many results still missing, please put them there!
  - The results which are there are quite conclusive:
    - A* with landmarks heuristic better than A* with straight-line heuristic
    - But not much better, at most a factor of 2

Your experiences with Ex. Sheet 1

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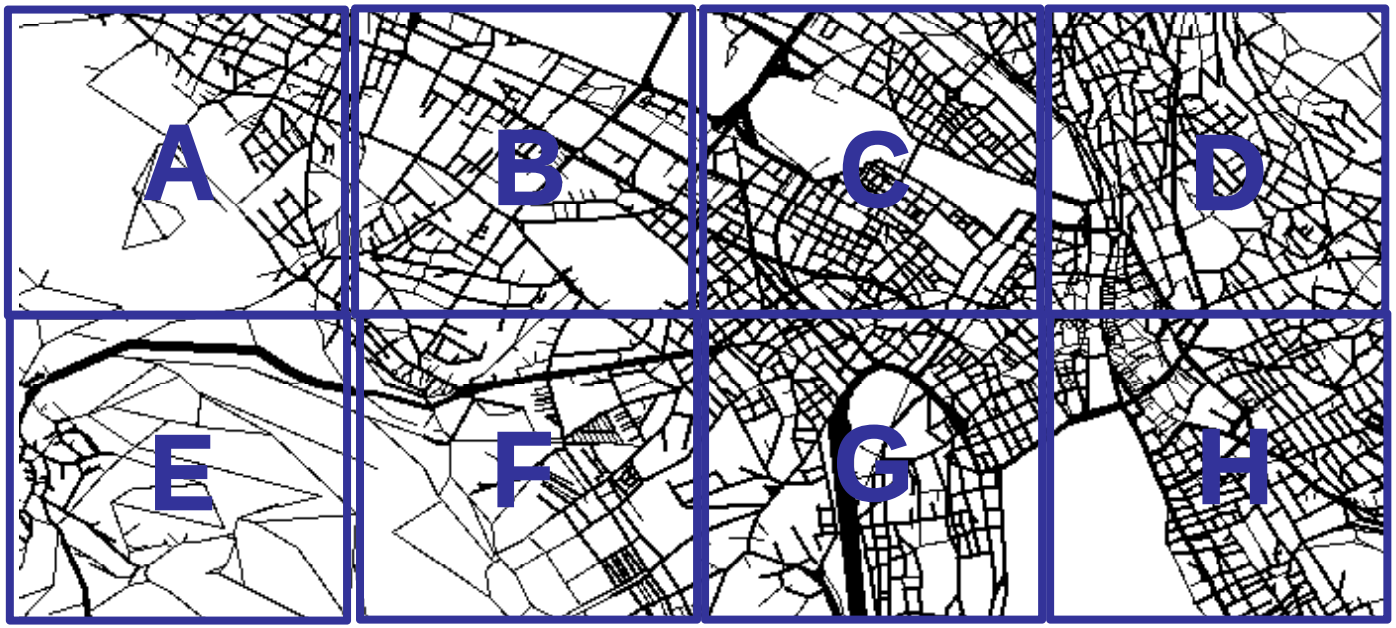
- Summary / excerpts
  - [can't access etna / stromboli right now :-(]

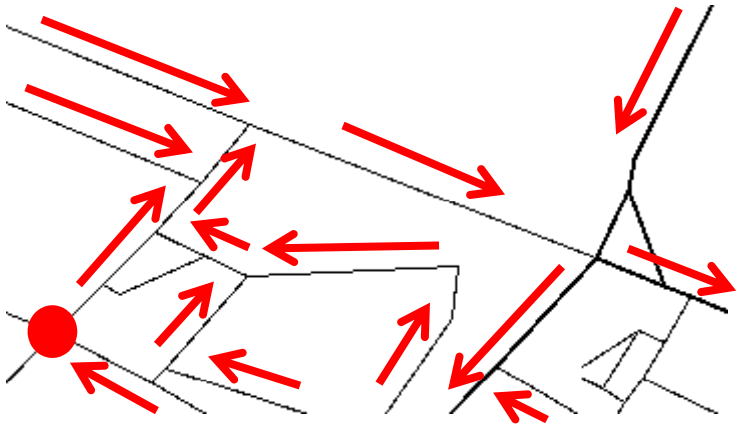
Arc flags — Basic idea 1/2

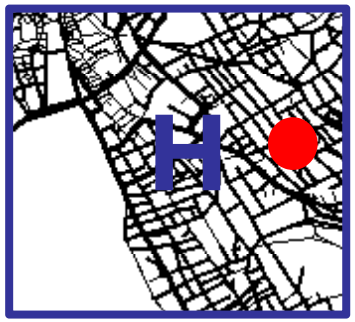
- Precomputation
  - Divide the map into "compact" regions of about equal size
  - For each arc, compute "direction signs" for each region
  - We call these direction signs **arc flags**



Arc flags — Basic idea 2/2

- At query time
  - Determine the region containing the target node
  - In Dijkstra's algorithm, outside of that region, consider only arcs which direction signs towards that regions

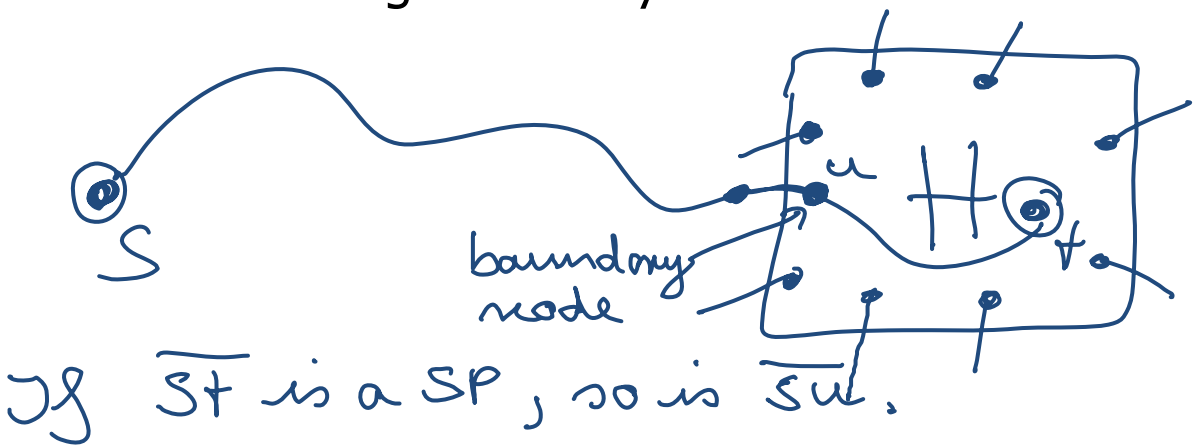




Arc flags — Details

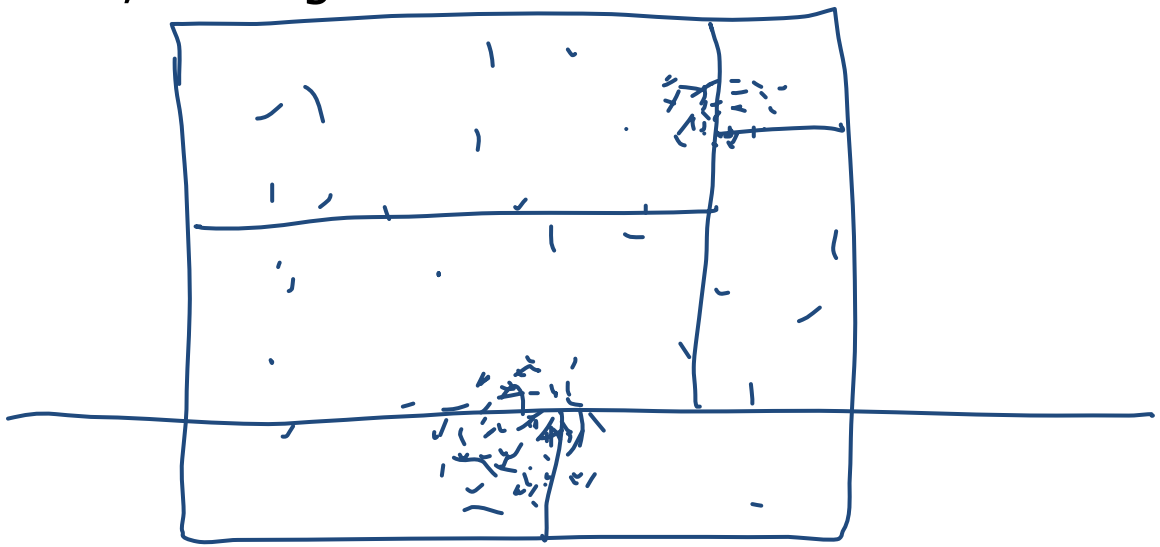
- How to do the precomputation
  - Consider a fixed region
  - For each arc we need to compute whether it is on a shortest path with target node in that region
  - We could compute all shortest paths to **each** node in that region **...** that would be **very** expensive
  - It suffices to compute shortest path to each **boundary node** of that region **...** why?

boundary node = node $\in H$ with arc to node $\notin H$



Arc flags — Details

- Compact regions
  - We call a region "compact" if it has few boundary nodes
  - Dividing a graph into k subgraphs of similar sizes with a minimal number of boundary nodes is a hard problem (graph partitioning)
  - For the exercise sheet, rectangular reasons are ok



Arc flags — Analysis 1/2

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- Space consumption
  - Assume we have k regions
  - Then we need k bits per arc for the direction signs
    - That is $k/8 \cdot m$ Bytes, where m = #arcs
  - Let's compare that to the storage needed for the graph
    - 12 bytes per node (OSM id + latitude + longitude)
    - 8 bytes per arc (head node id + cost)
    - That is $12n + 8m$ bytes, where n = #nodes, m = #arcs
    - For road networks we have $m \approx 2.25n$
    - That is, we need about 13 bytes / arc for the graph
    - So with $k > 100$ the arc flags start to become expensive

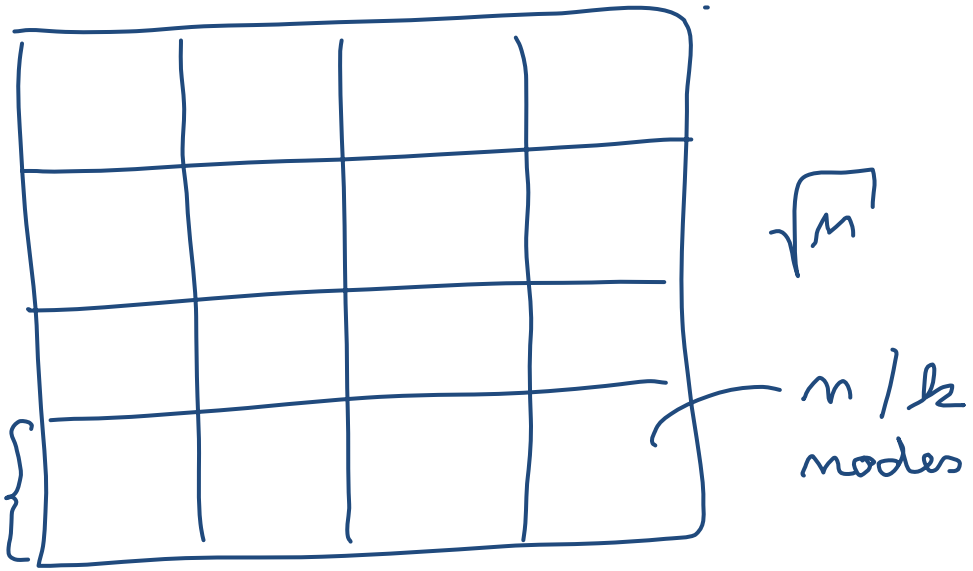
Arc flags — Analysis 2/2

- Precomputation time
  - We need a full Dijkstra computation for each node on the boundary of each region
  - How many boundary nodes do we have in total?

$4\sqrt{n/k} \cdot k$

$= 4\sqrt{nk}$

$\geq 4\sqrt{n}$



for $n = 1M$

$4\sqrt{n} = 4000$

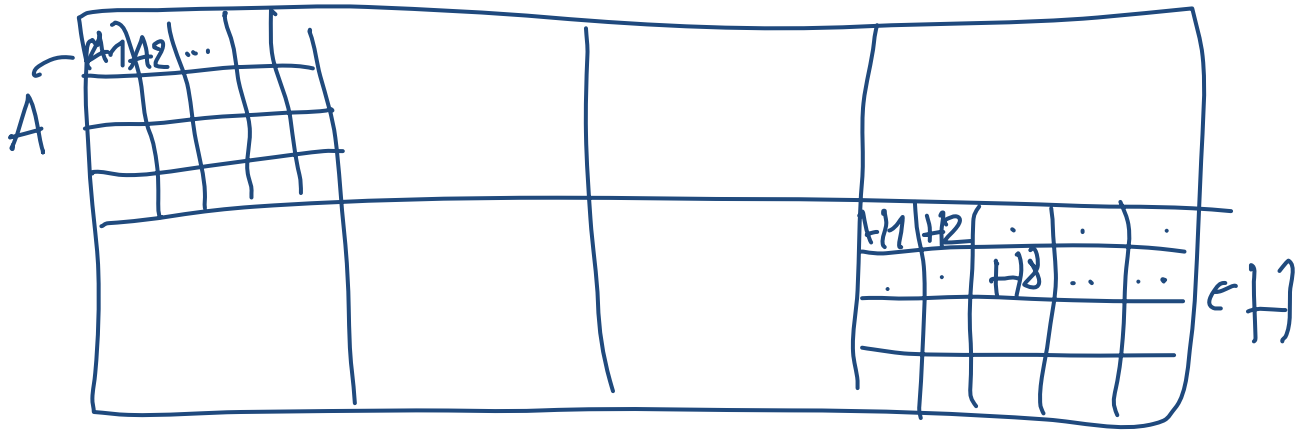
$n = 1M$, $k = 100$

$\Rightarrow 4\sqrt{nk} = 40K$

$\sqrt{n}$

Arc Flags — Hierarchical Approach 1/4

- We could have a hierarchy of regions
  - For the sake of explanation, let us assume two levels
  - Here is a picture:



Arc Flags — Hierarchical Approach 2/4

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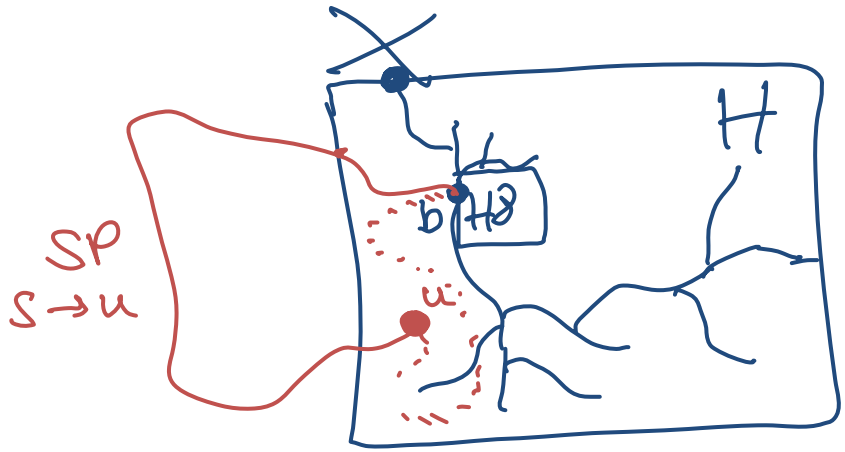
- Precomputation
  - Consider a target node in a small region H8, which is contained in big region H
  - Outside of H we can use signs to region H
  - We need to compute signs to region H8 only within H

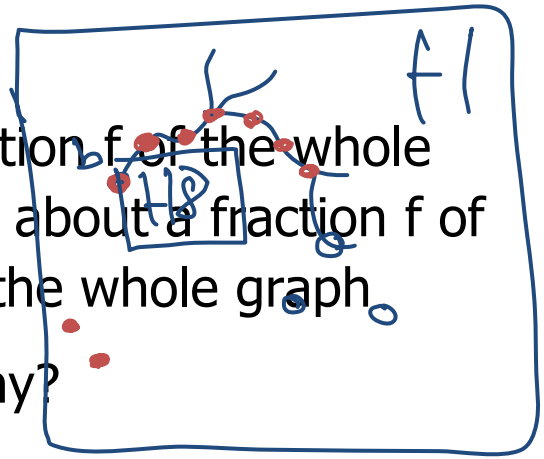
Arc Flags — Hierarchical Approach 3/4

- Precomputation Idea 1
  - Consider a fixed boundary node b of small region H8
  - We want to compute shortest paths from all nodes in H
  - So do a Dijkstra from b in the reverse graph
  - And ignore all nodes outside of H, that is, do not put them into the priority queue
  - This is WRONG ... why?



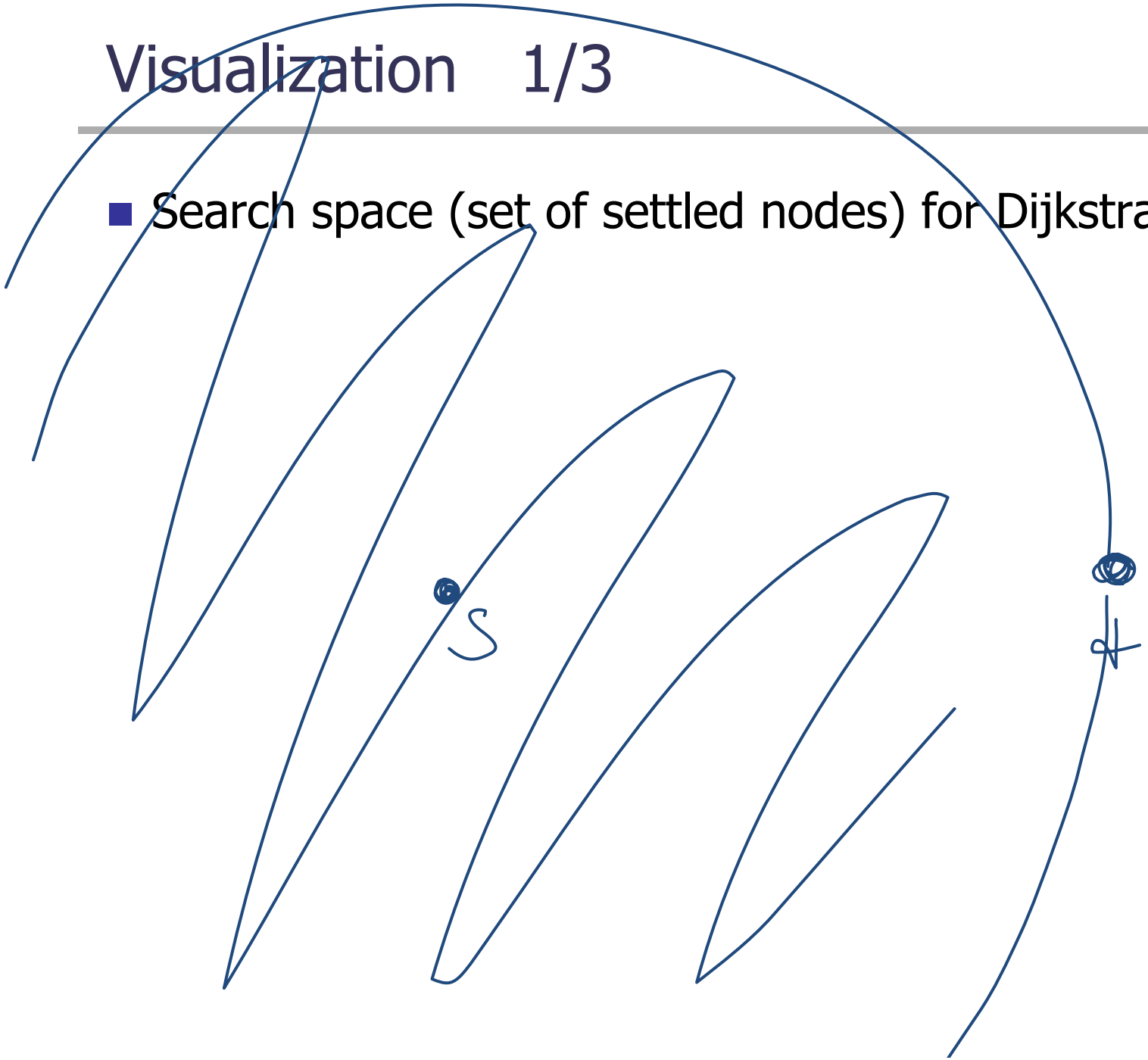
Arc Flags — Hierarchical Approach 4/4

- Precomputation Idea 2
  - Consider a fixed boundary node b of small region H8
  - We want to compute shortest paths from all nodes in H
  - So do a Dijkstra from b in the reverse graph
  - Stop when all nodes in H are settled
  - This is obviously correct
  - The hope is that if H contains a fraction f of the whole graph, than the running time is also about a fraction f of the running time of the Dijkstra on the whole graph
  - Unfortunately, that is not true ... why?



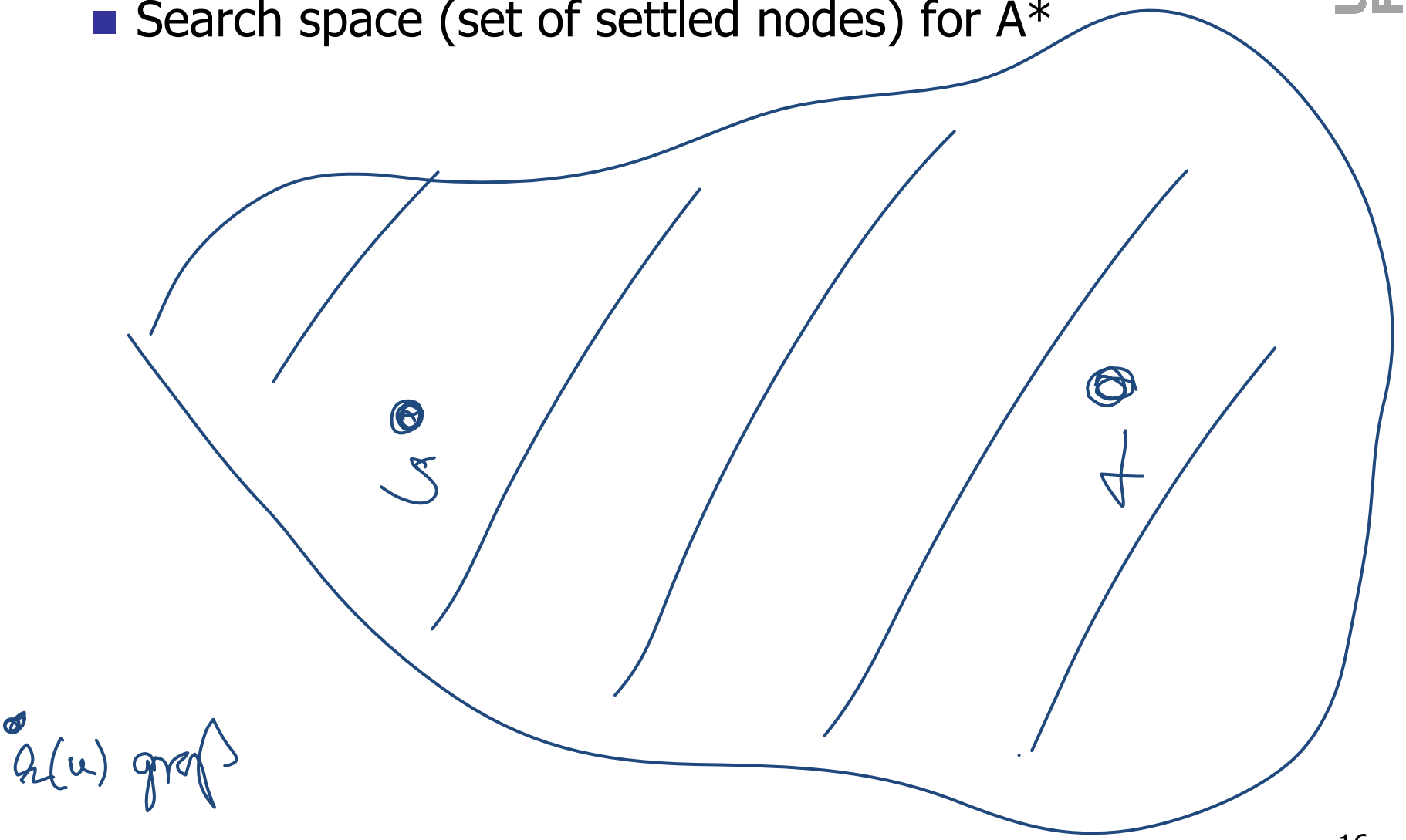
Visualization 1/3

- Search space (set of settled nodes) for Dijkstra



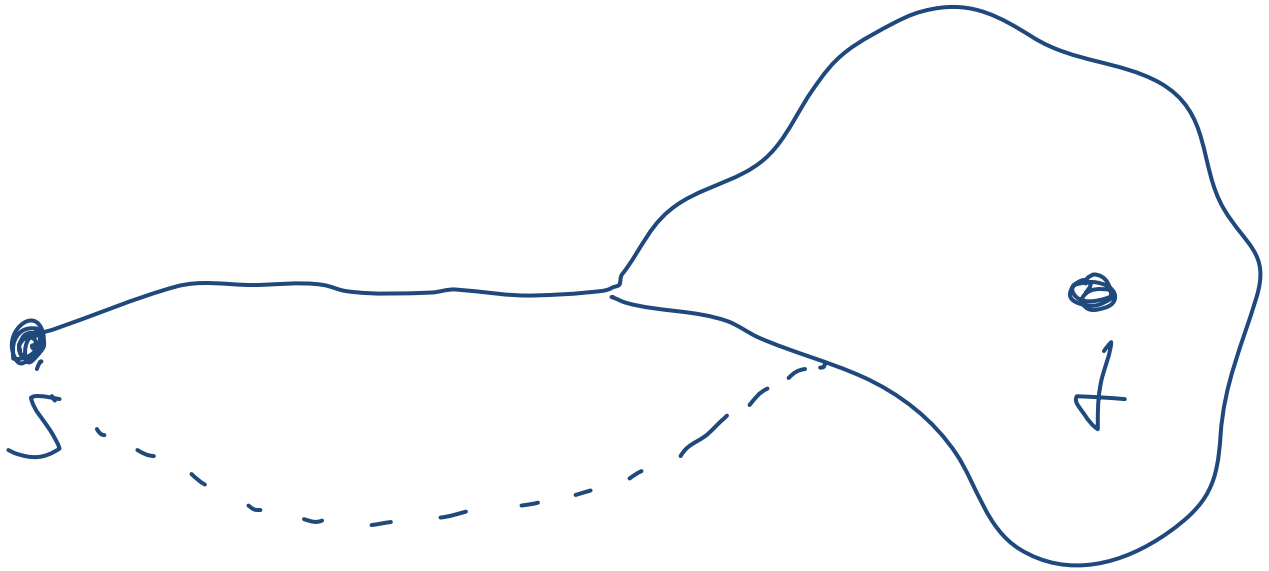
Visualization 2/3

- Search space (set of settled nodes) for A*



Visualization 3/3

- Search space (set of settled nodes) for Arc Flags



References

- First arc flag paper

  An extremely fast, exact algorithm for finding shortest paths in static networks with geographical background

  Ulrich Lauther, Münsteraner GI-Tage 2004

  https://gor.uni-paderborn.de/Members/AG06/LAUTHER.PDF

- Arc flags with various tricks + a hierarchy of regions

  Acceleration of Shortest Path and Constrained Shortest Path Computation

  E. Köhler and R. Möhring and H. Schilling, WEA 2005

  ftp://ftp.math.tu-berlin.de/pub/Preprints/combi/Report-042-2004.pdf

  http://www.springerlink.com/content/wc06qawxy5bc5bj0/