Search Engines

WS 2009 / 2010

Lecture 5, Thursday November 19th, 2009
(Efficient List Intersection)

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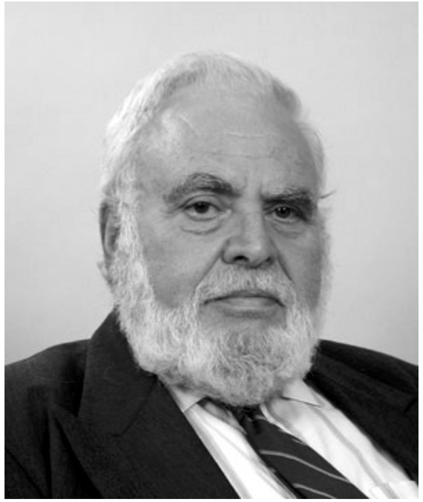
Overview of today's Lecture

- List intersection
  - you already know that it is at the core of what every search engine does

    (no fast search without fast list intersection)
  - revisit standard (linear-time) algorithm
  - prove a lower bound (how fast can we get)
  - algorithm that achieves a matching upper bound
- But before that
  - do missing proof for Golomb encoding from the last lecture
  - talk a bit about the exercises (again)

Golomb encoding (again)

- By Solomon Golomb, 1966
  - comes with a parameter M (modulus)
  - write positive integer x as $q \cdot M + r$
  - where $q = x \text{ div } M$ and $r = x \bmod M$
  - the codeword for x is then the concatenation of
    - the quotient q written in unary with 0s
    - a single 1 (as a delimiter)
    - the remainder r written in binary
  - examples



Solomon Golomb
*1932 Maryland

Golomb Encoding — Analysis

- Show that Golomb encoding is optimal
  - for gap-encoding inverted lists
  - assuming the doc ids in a list of size m are a random subset of size m of all doc ids $1..n$

[the proof]

On the exercises

- Amount
  - Should be less work this time
  - Do you prefer theoretical or practical or a mix?
- About the aspect of well-specifiedness
  - I am aware that the exercises are often not fully specified
  - this requires two things from your side
    - apply your common sense
    - in the case of doubt ask (intelligently)
  - these are two super-important skills to learn
    - real-life (research) problems are always ill-specifed (and actually much worse than in the exercises!)
    - common sense + communication are a must

List Intersection — Standard Algorithm

- For two lists A, B of sizes n and m:

[pseudocode of standard algorithm]

Improving the Standard Algorithm

- Two "engineering" problems with the previous code

1. there is an if-statement for each iteration of the loop

   why is that a problem?

   - modern processors do pipelining = execute future instruction while current instruction is not yet finished
   - in the case of an if, the processor tries to predict which part gets executed (so-called branch prediction)
   - if the prediction fails, the speculative execution of the future instructions has to be rolled back

2. the if-condition is also quite complex

   - costly to evaluate that in each iteration

List Intersection — Improved Version

- This code is (or can be) significantly faster:

[pseudocode of improved algorithm]

List Intersection — Lower Bound

- Can we do better than order n + m ?
  - if we want to compute the *union* of the two lists, we obviously can not
    - we have to output n + m elements in any case
  - for intersection we obviously can for special cases
    - for example: largest element of one list is smaller than smallest element of other list
    - then we can tell after one comparison that the intersection is empty
  - how about the general case
- Problem from now on
  - "locate" element from one list in the other list

List Intersection — Improvement 1

- Let's first try to improve on the standard algorithm
  - what if one list is much smaller than the other list?
    - length of smaller list is k
    - length of larger list is m
  - then we can binary search each element of the smaller list in the larger list

[an illustration of this]

  - complexity is $\sim k \cdot \log_2 m$
  - this is obviously better than $k + m$ when $k << m$

List Intersection — Improvement 2

- Simple observation:
  - if the previous element has been located at position i, the next binary search need only look at positions $\geq i$

[an illustration of this]

- Does this help us?
  - in the best case: [short calculation]
  - in the worst case: [short calculation]
  - in the "average" case: [short calculation]

List Intersection — Lower Bound

- Recall the lower bound for sorting n integers
  - there are $n!$ possible outputs
  - the algorithm has to distinguish between all of them
  - each comparison distinguishes between two cases
  - hence we need at least $\log_2 n!$ comparisons
  - by Stirling's formula $(n/e)^n \leq n! \leq n^n$
  - hence $\log_2 n! \sim n \cdot \log n$
  - hence every comparison-based sorting algorithm has a running time of $\Omega(n \cdot \log n)$
  - Note: not true for non-comparison based algorithms:

[explain by example 0-1 sequence]

List Intersection — Lower Bound

- Let's try the same for merging two lists A and B
  - that is, locate each element from A in B
  - again let k and m = number of elements in A and B resp.
  - same argument: how many ways are there to locate the k elements from A in the m elements from B
  - observe: each such way corresponds to a tuple $(i_1, \ldots, i_k)$ where $0 \leq i_1 \leq \ldots \leq i_k \leq m$

    (i_j is simply the location of the j-th element of A in B, location 0 means before the first element, location $i > 0$ means after the i-th element)
  - how many such tuples are there?

List Intersection — Lower Bound

- There is a similar quantity which is easy to count
  - the number of tuples $(i_1,\ldots, i_k)$ where $1 \leq i_1 < \ldots < i_k \leq n$
  - this is just the number of size-k subsets of $\{1,\ldots,n\}$
  - and the number of those is n over k $= n! / (k! \cdot (n-k)!)$
  - which by Stirling's formula is approximately $(e \cdot n/k)^k$

[relate the two kinds of quantities + prove the lower bound]

List Intersection — Lower Bound

List Intersection — Matching Upper Bound

- Idea:
  - after previous element from A has been located in B
  - start search from that location
  - but try to search not much further than next location

[an illustration of this]

  - trick: first *exponentional* search, then binary search
  - if the difference from the previous to the next location is d, this can be done in time O(d)

Analysis of this algorithm

- Terminology
  - Let $d_1, \ldots, d_k$ be the gaps between the locations of the k elements of A in B

    (d_1 = from beginning to first location)
  - note that $\Sigma_i\ d_i \leq m$ = number of elements in B
  - then the time complexity of the algorithm is $O(\Sigma_i \log d_i)$

[derive upper bound in terms of k and m]