Chair for Algorithms
and Data Structures
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Search Engines WS 09/10

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Exercise Sheet 11 — Solutions

Exercise 1 (Marjan)

TODO.

Exercise 2 (Marjan)

TODO.

Exercise 3 (Marjan)

TODO.

Exercise 4 (Hannah)

Since the events are independent, we have $L = \prod_{i=1}^{k} p_i^{n_i}$. Instead of L, we will minimize $\ln L = \sum_{i=1}^{k}(n_i \cdot \ln p_i)$, which gives the same result (because ln is a strictly monotone function) but is easier to deal with from the point of view of computing derivatives.

Now we want to find those $p_1, \ldots, p_k$ with sum 1 such that $\ln L$ is maximized. To compute the local optimae, we use Lagrangian optimization, as presented in the lecture, and we write:

$$\hat{L} = \sum i = 1^k (n_i \cdot \ln p_i) + \lambda \cdot (1 - \sum_{i=1}^{k} p_i).$$

Now set the partial derivatives with respect to each of λ, $p_1, \ldots, p_k$ to zero. Derivation by λ gives us the side constraint again, as usual:

$$\frac{\delta \hat{L}}{\delta \lambda} = 1 - \sum_{i=}^{k} p_i = 0.$$

Derivation by p_i gives us

$$\frac{\delta \hat{L}}{\delta n_i} = n_i / p_i - \lambda = 0.$$

Hence all n_i/p_i are equal, which means $p_i = C \cdot n_i$ for some constant C, and since the p_i have to sum to 1, we have $C = 1/n$, and hence $p_i = n_i/n$.

It remains to show that we have a local maximum at $p_i = n_i/n$, for $i = 1, \ldots, k$. The value of $\hat{L}$ at this location is $\sum_{i=1}^{k}(n_i \cdot \ln(n_i/n)) > 0$. On the border we have $p_i = 1$ for one i, and all other $p_j = 0$. The value of $\hat{L}$ is then $\ln 0 = -\infty$, and therefore our local optimum, which has a positive value, must be a local maximum.