Chair for Algorithms
and Data Structures
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Search Engines WS 09/10

http://ad.informatik.uni-freiburg.de/teaching

Exercise Sheet 10

complete until Thursday, January 21st

Exercise 1

Compute the singular value decomposition of the following 2×3 matrix *by hand.*

$$\begin{pmatrix} 3 & 5 & 5 \\ 3 & 7 & 1 \end{pmatrix}$$

Exercise 2

Let A_k be the the best rank-k approximation of A in the Frobenius norm, as defined in the lecture. Prove that $||A - A_k||_F^2 = \sigma_{k+1}^2 + \cdots + \sigma_m^2$, where σ_i is the ith largest singular value of A, and $m = rank(A)$. First try to prove this on your own for some time, and then feel free to ask for hints.

Exercise 3

Let A be a symmetric $m \times m$ matrix with m different, positive eigenvalues. The so-called *power method* starts with a random vector x and repeatedly applies A to it, and then normalizes the result after each step. That is, after k steps, it has computed $x_k = A^k \cdot x / ||A^k \cdot x||$. Prove that the power method converges to the eigenvector u_1 pertaining to the largest eigenvalue λ_1, that is, $\lim_{k\to\infty} x_k = u_1$.

Hint: Write x as a linear combination of the m eigenvectors $u_1, \ldots, u_m$ of A. You may assume without further proof that, because x is chosen at random, it is not orthogonal to u_1.

Exercise 4

Implement the power method for an arbitrary given symmetric matrix (it's ok to specify the matrix in the code). Don't use a library for matrix-matrix and matrix-vector multiplication but implement it yourself, it's simple. Run the method for a reasonable number of iterations. Check the correctness of your code by applying it to the 2×2 matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ from the lecture. For that matrix, $u_1 = (1/\sqrt{2}, 1/\sqrt{2})$ with eigenvalue 3.