

Exercise Sheet 6 — Solutions

Exercise 1

TODO.

Exercise 2

TODO.

Exercise 3

TODO.

Exercise 4 (Hannah)

Define $f(x) = x^2/2 + x - (1+x) \cdot \ln(1+x)$. We will show that $f(x) \geq 0$ for all $x \geq 0$. Then also $f(x)/x = x/2 + 1 - (1+1/x) \cdot \ln(1+x) \geq 0$ for $x > 0$, which proves the claim from the exercise for $x > 0$. It's easy to verify that it also holds for $x = 0$.

We can easily compute the derivative $f'(x) = x + 1 - \ln(1+x) - 1 = x - \ln(1+x)$. Since $1+x \leq e^x$ for all x , we have $f'(x) \geq 0$ for all x , that is, f is monotonically increasing over its whole range. Since $f(0) = 0$ that implies $f(x) \geq 0$ for all $x \geq 0$.